A distance-based topological relation model between spatial regions

Article *in* Arabian Journal of Geosciences · December 2018

DOI: 10.1007/s12517-018-4146-6

3 authors:

jw Shen
Southwest University in Chongqing
24 PUBLICATIONS **44** CITATIONS

Min Chen
Key Laboratory of Virtual Geographic Environment, Ministry of Education of PRC
86 PUBLICATIONS **827** CITATIONS

Songshan Yue
Nanjing Normal University
24 PUBLICATIONS **164** CITATIONS

Arabian Journal of Geosciences (2018) 11:763
https://doi.org/10.1007/s12517-018-4146-6

ORIGINAL PAPER

A distance-based topological relation model between spatial regions

Jingwei Shen[1] · Min Chen[2,3,4] · Songshan Yue[2,3,4]

Received: 25 April 2018 / Accepted: 30 November 2018

Abstract

Although the definitions of formal models used to represent spatial relations have gained increasing attention over the past 30 years, the linkage between topology and distance has not yet been effectively established. A topological relation model called the distance-based topological relation model (D-TRM) that considers both the topology and distance of spatial regions is proposed. The D-TRM is divided into two subtypes: the actual DTRM (AD-TRM) and the signed DTRM (SD-TRM). The actual distance is based on the distance in a two-dimensional space. The signed distance is based on the sign of the actual distance. Eight topological relations, namely, *disjoint*, *meet*, *overlap*, *cover*, *contain*, *equal*, *coveredBy* and *inside*, represented by the AD-TRM and SD-TRM are shown. The mutual exclusiveness among these eight topological relations represented by the SD-TRM is proven. The topological relation representations from the 9-intersection model (9IM), the splitting measures of the 9IM (SP-9IM), the SD-TRM and the AD-TRM are discussed, and the interoperability of each of the above models is summarised. The topological relation representation between the AD-TRM and the comprehensive set of 11 metric refinements is compared. The results show the following: (1) as the generalisation of the AD-TRM, the SD-TRM can concisely represent the topological relations; (2) the topology and distance between two spatial regions can be represented by the AD-TRM in a unified framework; (3) the AD-TRM provides a greater level of detail than the 9IM and (4) the D-TRM can express more distance information than the comprehensive set of 11 metric refinements.

Keywords Topological relation · Metric refinements · Distance-based topological relation model (D-TRM) · Spatial region

Introduction

Spatial relations are the relations between objects involving their relative position (Freemana 1975). Spatial relations are widely used to support the design of suitable query languages for spatial data retrieval and analysis in spatial databases and geographical information systems (GIS) (Schneider and Behr 2006). Spatial relation is commonly grouped into topological relations, direction relations and metric relations (Worboys 1992; Sharma 1996). Topological relations are invariant under topological transformations, such as translation, scaling and rotation (Egenhofer 1989). The integration of topological relation and other spatial relations is the current research focus (Bruns and Egenhofer 1998; Dong 2005; Nedas and Egenhofer 2008; Kor and Bennett 2013; Dube 2017). Topology is considered to be first-class information, whereas metric properties, are used as refinements that are frequently less exactly captured (Egenhofer and Mark 1995). Metric properties focus on the distance, length, perimeter and so on (Worboys 1992). Topological and metric are not totally separate concepts (Egenhofer and Dube 2009). Topological relation models that take metric properties into account can

✉ Songshan Yue
yss123yss@126.com

Jingwei Shen
sjwgis@swu.edu.cn

Min Chen
chenmin0902@163.com

1 Chongqing Engineering Research Center for Remote Sensing Big Data Application, School of Geographical Sciences, Southwest University, Chongqing 400715, China

2 Key Laboratory of Virtual Geographic Environment (Ministry of Education), Nanjing Normal University, Nanjing 210046, Jiangsu Province, China

3 State Key Laboratory Cultivation Base of Geographical Environment Evolution (Jiangsu Province), Nanjing Normal University, Nanjing 210023, China

4 Jiangsu Center for Collaborative Innovation in Geographical Information Resource Development and Application, Nanjing Normal University, Nanjing 210023, China

Published online: 08 December 2018

provide more details than pure topological relation models alone. Distance property, which is an important aspect of metric properties, can be used to quantitatively represent the degree of the proximity between spatial objects. An important present-day research trend is the integration of topological representations with those of distance.

Over the past three decades, many efforts have been made to formally define topological relations, for example, interval-based temporal logic, region connection calculus (RCC) and point-set topology. Interval-based temporal logic is suitable for expressing topological relations in a one-dimensional space. For example, interval-based temporal logic has been proposed to express the topological relations of temporal intervals (Allen 1983; Hornsby et al. 1999; Egenhofer 2007). The core of the RCC approach is to consider whether there is a connection between spatial objects. The RCC approach has been widely utilised for qualitative topological representations and for reasoning between regions (Randell and Cohn 1989; Randell et al. 1992; Cohn et al. 1995, 1997; Gotts et al. 1996; Jonsson and Drakengren 1997; Stell and Worboys 1997; Li and Li 2006). The RCC-5 and RCC-8 schemes consist of five and eight basic topological relations, respectively. Meanwhile, the core of the point-set topological relation model is to consider whether the intersections between the subsets of spatial objects are empty. Research has been conducted on the formalisation of topological relations based on the point-set topology relation model (Egenhofer and Franzosa 1991, 1995; Egenhofer and Herring 1991; Egenhofer et al. 1993, 1994; Clementini et al. 1993; Open GIS Consortium 1999; Chen et al. 2001; Deng et al. 2007; Liu and Shi 2007; Kurata 2009; Alboody et al. 2010; Leng et al. 2017; Shen et al. 2017; Formica et al. 2017). The 9-intersection model (9IM) is a well-known comprehensive model for topological spatial relations based on point-set topology and 512 binary topological relations can be distinguished accordingly. The RCC-8 and 9IM topological relation models can effectively represent topological relations, whereas the lack of metric properties in these two models will seriously affect the level of detail regarding the representation of topological relations. The following two examples motivate the interest in the integration of topological relation and metric properties. First, California and Colorado are disjoint, and California and Maine are disjoint. Although equipped with the same topological relation, the distance from California to Colorado is less than the distance from California to Maine. Second, although an island is always inside a lake in the whole year, the distance from the island to the boundary of the lake may changes in different seasons. If only topological relation is considered, differences in distance will not be distinguished.

Metric refinements have been applied to enhance topological relations. Topological relation models that incorporate metric properties can provide a greater level of detail than a purely topological relation model alone. Thus, various approaches that consider metric refinements of topological relations have been widely investigated (Egenhofer and Shariff 1998; Shariff et al. 1998; Godoy and Rodríguez 2002; Nedas et al. 2007; Egenhofer and Dube 2009; Sridhar et al. 2011; Dube et al. 2015; Penna et al. 2017). Metric properties, such as splitting ratios, closeness measures, expansion closeness, contraction closeness and approximate alongness, are discussed in the above literatures. Compared with those of topological relation models, metric refinements of topological relations can describe both topological relations and metric relations. Although many other metric properties have been integrated into topological relation models, the linkage between topology and distance has not been established effectively. Due to the absence of distance information in these models, the movement of the spatial object cannot be expressed by these topological relation models. For example, a small translation of one object usually does not modify the topological relations between two spatial objects; however, the distance between spatial objects has changed. The absence of distance information has seriously affected the accuracy of spatial relations.

Let us move on to the distance that is an important aspect of spatial relations, and many distance models have been proposed independently of topological relation models or order relation models. The distance properties in some models are continuous. For example, a fuzzy membership function, which denotes the degree of membership (between zero and one, inclusive), was constructed (Dutta 1989). The distance properties in many other models are discrete. For example, a method to rank distances into a number of intermediate steps from 0 to $n-1$ was proposed (Frank 1992). In addition, a model with the characteristic advantage of flexible sets of distance distinctions at various levels of granularity for the qualitative representation of distances in the context of a geographic space was developed (Hernández et al. 1995), fuzzy sets were used to describe the closeness between various objects (Guesgen 2002), distance relations within a particular environmental space were discussed (Worboys 2001) and the qualitative spatial relation nearness (Duckham and Worboys 2001) was explored. Following these ideas, many research studies have considered both continuous and discrete approaches for the representation of distance metrics. For example, one model was developed to consider both absolute and relative methods for judging distances (Gahegan 1995) and another was presented to determine the qualitative distance between two features through a network (Schultz et al. 2007). Accordingly, qualitative and quantitative models are widely used to represent spatial distance relations.

Following the precondition that “topology matters, metric refines” (Egenhofer and Mark 1995), this paper presents a distance-based topological relation model (D-TRM). Unlike models based on RCC or point-set topology, connections or intersections are not considered and distance is taken into account in the D-TRM. The advantage of the D-TRM lies in two aspects: (1) the D-TRM is concisely developed to consider only four elements and (2) both topological relations and distances between spatial regions can be expressed in a unified framework. The queries, such as "Whether the topological relation between California and Colorado are equal to the topological relation between California and Maine or not? Whether the distance from California to Colorado is nearer than the distance from California to Maine or not?" and "What is the distance and topological relation of the island and the lake?" will be easily addressed by the D-TRM.

The remainder of this paper is outlined as follows. Section "The distance-based topological relation model" introduces the proposed topological relation model for spatial regions. Section "Results" describes the eight topological relations represented by the proposed model and the mutual exclusion of the eight topological relations. Section "Discussion" discusses and compares the proposed model with other models. Section "Conclusions" provides the conclusions of this study and discusses future work.

The distance-based topological relation model

In this section, the D-TRM for spatial regions is proposed. Both the topology and the distance between two spatial regions are represented concisely by the D-TRM in a unified framework. According to the degree of refinement of the distance, two subtypes of the D-TRM, i.e. the actual D-TRM (AD-TRM) and the signed D-TRM (SD-TRM), are given to represent the actual distance and the sign of the distance, respectively.

The actual distance-based topological relation model

To describe a topological relation model, the properties of a spatial region, including its interior, boundary and exterior, should be defined first. The definitions of these properties are given below.

Definition 1:

A spatial region is defined as a connected interior, a connected exterior and a connected boundary (Egenhofer and Herring 1991; Egenhofer and Sharma 1993). The interior of the region is the union of all open sets that are contained in the region, the boundary of a region comprises a number of lines, and the exterior of a region is the set that does not contain the region.

Let A and B represent two spatial regions. The AD-TRM, which consists of four parts, is represented as

$$R_{\mathrm{AD-TRM}}(A,B)=\begin{bmatrix}\mathrm{Min}(\partial A,\partial B) & \mathrm{Max}(\partial A,\partial B)\\ \mathrm{Min}(\partial B,\partial A) & \mathrm{Max}(\partial B,\partial A)\end{bmatrix} \quad (1)$$

where $\partial A$and ∂B are the boundaries of the spatial regions A and B, respectively. The AD-TRM can also be represented as a four-tuple, i.e. $\mathrm{Min}(\partial A,\partial B)$, $\mathrm{Max}(\partial A,\partial B)$, $\mathrm{Min}(\partial B,\partial A)$ and $\mathrm{Max}(\partial B,\partial A)$. $\mathrm{Min}(\partial A,\partial B)$ is the minimum distance from ∂A to ∂B, $\mathrm{Max}(\partial A,\partial B)$ is the maximum distance from ∂A to ∂B, $\mathrm{Min}(\partial B,\partial A)$ is the minimum distance from $\partial B$to ∂A and $\mathrm{Max}(\partial B,\partial A)$ is the maximum distance from ∂B to ∂A. The value of the tuple in the AD-TRM can be positive, negative or zero. The definitions of these positive, negative and zero distance values are given below.

Definition 2:

Let V and W be separate boundary points for the spatial regions A and B, respectively. If W is in the exterior of A, the distance between $V$and $W$is positive. If $W$is along the boundary of A, the distance between $V$and $W$is zero. If $W$is within the interior of A, the distance between $V$and $W$is negative.

In Fig. 1, $V$is a boundary point of the spatial region A, and $W1$–$W3$ are boundary points of the spatial region B. According to Definition 2, the distance between $V$and $W1$ is positive, the distance between $V$and $W2$ is zero, and the distance between $V$and $W3$ is negative. In Fig. 1, +, − and 0 are positive distance, negative distance and zero, respectively.

To acquire the minimum and maximum distances, the ordinal relations of the positive, negative and zero distances are defined below.

Definition 3:

A positive distance is larger than zero distance, and a zero distance is larger than a negative distance.

In Fig. 1, according to Definition 3, the distance between V and $W1$ is larger than the distance between $V$and $W2$, and the distance between $V$and $W2$ is larger than the distance between $V$and $W3$.

In the AD-TRM, the ordinal relation between the minimum distance and the maximum distance is defined below.

Definition 4:

The minimum distance is no greater than the maximum distance.

Because the minimum distance and the maximum distance are the minimum value and the maximum value between the

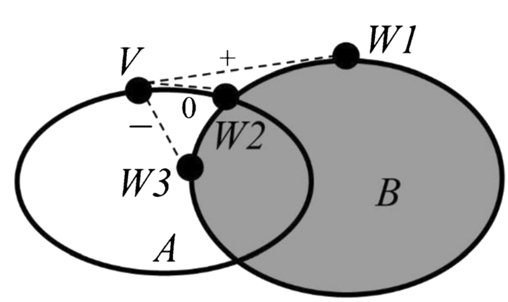


Fig. 1 Positive, negative and zero distances

two boundaries, respectively, the maximum distance is no less than the minimum distance.

Definition 5:

The minimum distance may be equal to the maximum distance. In this case, both the minimum distance and the maximum distance are equal to zero.

Because the boundaries of A and B coincide when A is equal to B, both the minimum distance and the maximum distance are equal to zero according to Definition 2.

The signed distance-based topological relation model

Compared with the AD-TRM, the SD-TRM uses the sign of the actual distance instead of the actual distance. The SD-TRM, which also consists of four parts, is represented as

$$R_{\mathrm{SD-TRM}}(A,B) = \begin{bmatrix} \mathrm{SD}(\mathrm{Min}(\partial A,\partial B)) & \mathrm{SD}(\mathrm{Max}(\partial A,\partial B)) \\ \mathrm{SD}(\mathrm{Min}(\partial B,\partial A)) & \mathrm{SD}(\mathrm{Max}(\partial B,\partial A)) \end{bmatrix} \tag{2}$$

where SD $(\mathrm{Min}(\partial A, \partial B))$ is the sign of the minimum distance from ∂A to ∂B, SD $(\mathrm{Max}(\partial A, \partial B))$ is the sign of the maximum distance from ∂A to ∂B, SD $(\mathrm{Min}(\partial B, \partial A))$ is the sign of the minimum distance from $\partial B$to ∂A and SD $(\mathrm{Max}(\partial B, \partial A))$ is the maximum distance from ∂B to ∂A. The SD-TRM can also be represented as a four-tuple, i.e. SD $(\mathrm{Min}(\partial A, \partial B))$, SD $(\mathrm{Max}(\partial A, \partial B))$, SD $(\mathrm{Min}(\partial B, \partial A))$ and SD $(\mathrm{Max}(\partial B, \partial A))$. The value of each element in the SD-TRM can be drawn from the set $\{-1, 0, 1\}$. A value of -1 means that the distance is negative, 0 denotes a zero distance and a value of 1 represents a positive distance. By considering values of $\{-1, 0, 1\}$, 81 ($81 = 3^4$) topological relations are possible between two spatial regions. However, not all of the 81 topological relations can be implemented; instead, only parts of the topological relations between two spatial regions can be implemented. In the following formulas, the value of an element that can take an arbitrary value is marked with "*". Based on Definitions 6 through 7, some impossible topological relations can be excluded.

Definition 6:

The sign of the minimum distance is no greater than the sign of the maximum distance.

$$R_{\mathrm{SD-TRM}}(A,B) \neq \begin{bmatrix} 1 & 0 \\ * & * \end{bmatrix} \vee \begin{bmatrix} 1 & -1 \\ * & * \end{bmatrix} \vee \begin{bmatrix} 0 & -1 \\ * & * \end{bmatrix} \vee \begin{bmatrix} * & * \\ 1 & 0 \end{bmatrix} \vee \begin{bmatrix} * & * \\ 1 & -1 \end{bmatrix} \vee \begin{bmatrix} * & * \\ 0 & -1 \end{bmatrix} \tag{3}$$

Proof: Because a positive distance is larger than zero, a zero distance is larger than a negative distance, and the maximum distance is larger than the minimum distance, the sign of the minimum distance is no greater than the sign of the maximum distance.

Definition 7:

It is impossible to set all elements equal to -1.

$$R_{\mathrm{SD-TRM}}(A,B) \neq \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \tag{4}$$

Proof: If both the minimum distance and the maximum distance from A to B are -1, then B is entirely located within the interior of A. Therefore, both the minimum distance and the maximum distance from A to B are greater than 0.

Results

Topological relations from the AD-TRM and SD-TRM

There are eight topological relations, including disjoint, meet, overlap, cover, contain, equal, coveredBy and inside, between two regions embedded within a two-dimensional space that are characterised by a single connected boundary. Figure 2

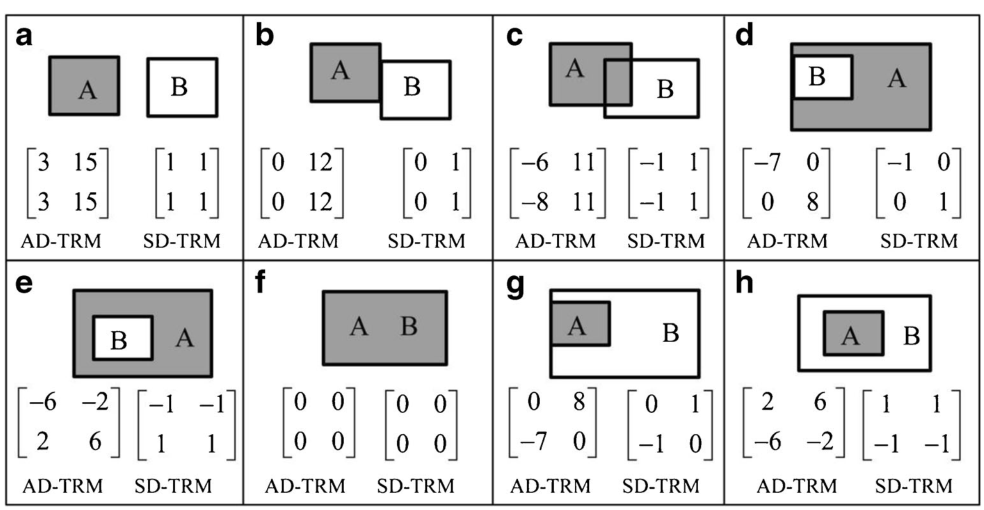


Fig. 2 The eight topological relations based on the AD-TRM and SD-TRM between two spatial regions. **a** Disjoint. **b** Meet. **c** Overlap. **d** Cover. **e** Contain. **f** Equal. **g** CoveredBy. **h** Inside

shows these eight topological relations based on the AD-TRM and SD-TRM.

Inference 1: If all four values are greater than 0, then the two regions are disjoint (Fig. 2a).

If all four values are greater than 0, an arbitrary point on the boundary of spatial region A is located in the exterior of spatial region B, and vice versa. The minimum and maximum distances from A to B are equal to the minimum and maximum distances from B to A, respectively. Therefore, Min $(\partial A, \partial B)$ = Min $(\partial B, \partial A)$ and Max $(\partial A, \partial B)$ = Max $(\partial B, \partial A)$ are true for the disjoint conditions between A and B. Likewise, SD (Min$(\partial A, \partial B)$) = SD (Min$(\partial B, \partial A)$) and SD (Max$(\partial A, \partial B)$) = SD (Max$(\partial B, \partial A)$) are true.

Inference 2: If the minimum distance from spatial region A to B is equal to 0, the maximum distance from A to B is greater than 0, the minimum distance from B to A is equal to 0 and the maximum distance from B to A is greater than 0, then the two regions meet (Fig. 2b).

For the meet relation, the minimum distance from spatial region A to B is equal to 0, and vice versa. Similar to the disjoint relation, the minimum and maximum distances from A to B are equal to the minimum and maximum distances from B to A, respectively.

Inference 3: If the minimum distance from spatial region A to B is less than 0, the maximum distance from A to B is greater than 0, the minimum distance from B to A is less than 0 and the maximum distance from B to A is greater than 0, then the two regions overlap (Fig. 2c).

For the overlap relation, the maximum distance from spatial region A to B and the maximum distance from B to A are equal. Taking Fig. 3 as an example, the maximum distance from A to B is 100, and the maximum distance from B to A is 100. However, the minimum distance from A to B and the minimum distance from B to A may be not equal. For example, Q1 is the boundary point of A and B, the planar distance between P and Q1 is 70, and the planar distance between Q1 and Q3 is 50. According to Definition 2, the distance from P to Q1 is 0, and the distance from Q3 to Q1 is 0. To calculate the minimum distance, the value of σ, which is positive and infinitely close to zero, is defined.

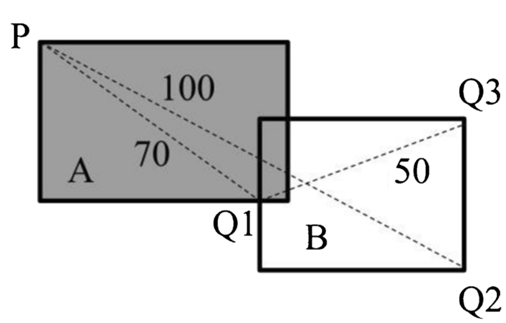


Fig. 3 Overlap relation and the minimum and maximum distances between spatial region A and B

The minimum distance from A to B is $-(70-\sigma)$, and the minimum distance from B to A is $-(50-\sigma)$.

Inference 4: If the minimum distance from spatial region A to B is less than 0, the maximum distance from A to B is equal to 0, the minimum distance from B to A is equal to 0 and the maximum distance from B to A is greater than 0, then A covers B (Fig. 2d).

For the cover relation, the interior of B is entirely within the interior of A, and A and B share some common boundary. Therefore, the maximum distance from A to B and the minimum distance from B to A are equal. However, the absolute value of the minimum distance from A to B may not be equal to the absolute value of the maximum distance from B to A.

Inference 5: If the minimum and maximum distances from spatial region A to B are less than 0, and if the minimum and maximum distances from B to A are greater than 0, then A contains B (Fig. 2e).

For the contain relation, B is entirely within the interior of A. The absolute value of the minimum distance from A to B is equal to the absolute value of the maximum distance from B to A; however, their signs are different. Likewise, the absolute value of the maximum distance from A to B is equal to the absolute value of the minimum distance from B to A; however, their signs are also different.

Inference 6: If all four values are equal to 0, then the two spatial regions are equal (Fig. 2f).

For the equal relation, the boundaries of A and B coincide. Therefore, the distance between any two points is always zero.

Inference 7: If the minimum distance from spatial region A to B is equal to 0, the maximum distance from A to B is greater than 0, the minimum distance from B to A is less than 0, and the maximum distance from B to A is equal to 0, then A is coveredBy B (Fig. 2g).

For the coveredBy relation, the interior of A is entirely within the interior of B, and A and B share some common boundary. Therefore, the minimum distance from A to B and the maximum distance from B to A are equal. However, the absolute value of the maximum distance from A to B may not be equal to the absolute value of the minimum distance from B to A.

Inference 8: If the minimum and maximum distances from spatial region A to B are greater than 0, and if the minimum and maximum distances from B to A are less than 0, then A is inside B (Fig. 2h).

For the inside relation, spatial region A is entirely within the interior of spatial region B. The absolute value of the minimum distance from A to B is equal to the absolute value of the maximum distance from B to A; however, their signs are different. Likewise, the absolute value of the maximum distance from A to B is equal to the absolute value of the minimum distance from B to A; however, their signs are also different.

Mutual exclusiveness of topological relations

The eight topological relations are mutually exclusive, that is, only one topological relation in the SD-TRM holds true for any two spatial regions in a given situation. Even with the same topological relations between two pairs of regions, the representation of their topological relations based on the AD-TRM may be different. Since the AD-TRM is a refinement of the SD-TRM, the value of the SD-TRM can be derived from the value of the AD-TRM and the mutual exclusiveness of topological relations defined by the SD-TRM can be proven. To prove the mutual exclusiveness of these topological relations, the topological relation decision tree, which is a tree-structured classification model, is introduced (Fig. 4).

In the topological relation decision tree, each internal node (black circles in Fig. 4) represents a test of an attribute, each branch represents the outcome of that test, and each leaf node (white circles in Fig. 4) represents a specific topological relation. The paths from the root to the leaf nodes represent classification rules. Because the value of each attribute can be − 1, 0 or 1, each internal node has three branches. If the attribute is − 1, the leftmost branch is followed. If the attribute is 0, the middle branch is followed. If the attribute is 1, the rightmost branch is followed. This process is repeated until the leaf node is reached. If the branch is a solid line, the value of the attribute is possible; otherwise, the value of the attribute is impossible (dotted lines in Fig. 4). There are three dotted lines in Fig. 4. Since the topological relation decision tree in Fig. 4 has only burst nodes but no sink nodes, the topological relation decision tree can prove the mutual exclusiveness of topological relations.

Take the leftmost dotted line (labelled as ① in Fig. 4) for example. Because the sign of the minimum distance is no greater than the sign of the maximum distance according to Definition 6, if the SD(Min(∂A, ∂B)) = 0, then it is impossible for SD(Max(∂A, ∂B)) = − 1.

For the middle dotted line (labelled as ② in Fig. 4), if SD(Min(∂A, ∂B)) = 0 and SD(Max(∂A, ∂B)) = 1, then it is impossible for SD(Min(∂B, ∂A)) = 1 (Formula (7)).

$$R_{\mathrm{AD-TRM}}(A,B) \neq \begin{bmatrix} 0 & 1 \\ 1 & * \end{bmatrix} \tag{7}$$

Proof: If SD (Min(∂A, ∂B)) = 0 and SD (Max(∂A, ∂B)) = 1, part of B's boundary coincide with part of A's boundary, and other part of B's boundary must be in the exterior of A. Only the coveredBy and meet relations meet these conditions. The value of SD (Min(∂B, ∂A)) is − 1 for the coveredBy relation and that of SD (Min(∂B, ∂A)) is 0 for the meet relation, and it is impossible for SD (Min(∂B, ∂A)) = 1.

For the rightmost dotted line (labelled as ③ in Fig. 4), if SD (Min(∂A, ∂B)) = 1 is true, then it is impossible for SD (Min(∂B, ∂A)) = 0 (Formula (8)).

$$R_{\mathrm{AD-TRM}}(A,B) \neq \begin{bmatrix} 1 & * \\ 0 & * \end{bmatrix} \tag{8}$$

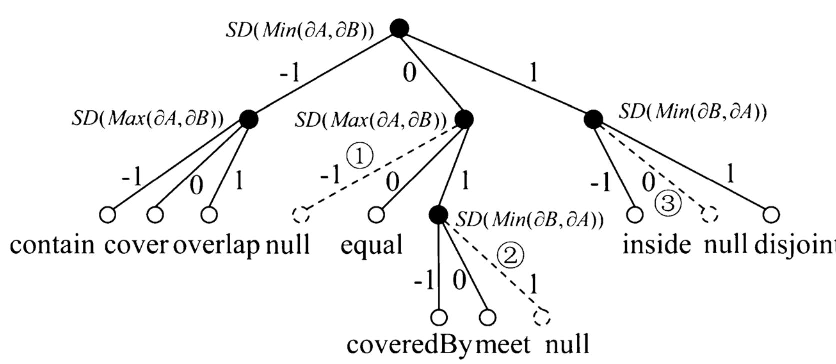
Proof: If SD (Min(∂A, ∂B)) = 1 is true, then spatial region A does not contain any boundary point of spatial region B, and only the disjoint and inside relations meet these conditions. The value of SD (Min(∂B, ∂A)) is − 1 for the inside relation and that of SD (Min(∂B, ∂A)) is 1 for the disjoint relation, and it is impossible for SD (Min(∂B, ∂A)) = 0.

Discussion

Topological relation representations using the 9IM, splitting measures of the 9IM, SD-TRM and AD-TRM

Many topological relation models have been proposed. Thus, it would be of interest to compare the proposed model with other models. The 9IM, which is a widely used topological relation representation model, is defined by Formula (9) (Egenhofer and Herring 1991). Nine splitting measures that offer refinement opportunities for the 9IM are defined as ratios (Formula (10)) (Egenhofer and Dube 2009). In Formulas (9–10), A^o, ∂A, A^-, B^o, ∂B and B^- are A's interior, A's boundary, A's exterior, B's interior, B's boundary and B's exterior, respectively. In Formula (10), the functions of area(), length() and

Fig. 4 The topological relation decision tree



bounded(), are to compute the area, compute the length and compute the area of a spatial object's exterior shut off by the union of the two spatial objects. Among these splitting measures, either the area of an intersection with respect to spatial region A or the length of an intersection with respect to the length of the boundary of A is used for the refinement of the empty or non-empty splitting measures of the 9IM (SP-9IM).

$$R_{9\mathrm{IM}}(A,B)=\begin{bmatrix} A^o\cap B^o & A^o\cap\partial B & A^o\cap \mathrm{B}^- \\ \partial A\cap B^o & \partial A\cap\partial B & \partial A\cap \mathrm{B}^- \\ \mathrm{A}^-\cap B^o & \mathrm{A}^-\cap\partial B & \mathrm{A}^-\cap \mathrm{B}^- \end{bmatrix} \tag{9}$$

$$R_{\mathrm{SP-9IM}}(A,B)=\begin{bmatrix} \frac{\mathrm{area}\,(A^o\cap B^o)}{\mathrm{area}\,(A)} & \frac{\mathrm{length}\,(A^o\cap\partial B)}{\mathrm{length}\,(\partial A)} & \frac{\mathrm{area}\,(A^o\cap B^-)}{\mathrm{area}\,(A)} \\ \frac{\mathrm{length}\,(\partial A\cap B^o)}{\mathrm{length}\,(\partial A)} & \frac{\mathrm{length}\,(\partial A\cap\partial B)}{\mathrm{length}\,(\partial A)} & \frac{\mathrm{length}\,(\partial A\cap B^-)}{\mathrm{length}\,(\partial A)} \\ \frac{\mathrm{area}\,(A^-\cap B^o)}{\mathrm{area}\,(A)} & \frac{\mathrm{length}\,(A^-\cap\partial B)}{\mathrm{length}\,(\partial A)} & \frac{\mathrm{area}\,(\mathrm{bounded}(A^-\cap B^-))}{\mathrm{area}\,(A)} \end{bmatrix} \tag{10}$$

Some instances are given in Fig. 5. Table 1 lists the results of comparisons between the 9IM, SP-9IM, SD-TRM and AD-TRM. ∞ is infinity in Table 1. Because the intersection of A's exterior and B's always is infinity in Fig. 5a–d.

The formalisms of the 9IM for Fig. 5a, b and for Fig. 5c, d are the same because it considers only empty and non-empty values, and the 9IM concisely represents topological relations. Although there are obvious distance differences in this figure, their formalisms of the 9IM are identical.

Likewise, the formalisms of the SP-9IM for Fig. 5a, b and for Fig. 5c, d are the same. This is due to the same shapes of A and B in Fig. 5a, b, the same topological relations in Fig. 5a, b, the same shapes of A and B in Fig. 5c, d, the same topological relations in Fig. 5c, d, and the same formalisms of the SP-9IM for Fig. 5a, b and for Fig. 5c, d. Although the SP-9IM, which is the refinement of the 9IM, can distinguish among nine splitting measures, it does not have the ability to represent the distance.

Similarly, the formalisms of the SD-TRM for Fig. 5a, b and for Fig. 5c, d are the same because it considers only the sign of the distance, and the SD-TRM concisely represents topological relations. Although there are obvious differences between the topological relations using the SD-TRM in this figure, their formalisms are identical. Both the SD-TRM and 9IM can concisely represent topological relations between spatial regions.

However, the formalisms of the AD-9IM for Fig. 5a, b and for Fig. 5c, d are different. This is because the AD-TRM considers the minimum and maximum distances between spatial regions and provides more details than the 9IM and SD-TRM.

The focuses of the AD-TRM and SP-9IM are different. The advantage of the AD-TRM is the emphasis on the distance, whereas the advantage of the SP-9IM is the emphasis on the splitting measures. The AD-TRM is the refinement of the SD-TRM, and the SD-TRM is the generalisation of the AD-TRM.

The interoperability of the AD-TRM, SD-TRM, 9IM and SP-9IM

The AD-STM, SD-TRM, 9IM and SP-9IM can effectively represent the eight topological relations between spatial regions. Interoperability implies an exchange between the different topological relation models. The transformation of the above mentioned four models are discussed in this section.

First, the interoperability of the AD-TRM and SD-TRM is investigated. Because the AD-TRM is a refinement of the SD-TRM, the value of the SD-TRM can be easily derived from the value of the AD-TRM. However, because the SD-TRM is the generalisation of the AD-TRM, it is impossible to infer the AD-TRM from the SD-TRM. If a_dis is the actual distance in the AD-TRM and s_dis is the signed distance in the SD-TRM, the interoperability of the AD-TRM with the SD_TRM is given by Formula (11). In Formula (11), a_dis can be any real number, whereas s_dis can only be −1, 0 or 1. As concluded from Formula (11), there is always a corresponding value of the SD_TRM for the value of the AD_TRM.

$$s\_dis=\begin{cases} 1 & \text{if } (a\_dis>0) \\ 0 & \text{if } (a\_dis=0) \\ -1 & \text{if } (a\_dis<0) \end{cases} \tag{11}$$

Second, the interoperability of the SD-TRM and 9IM is explored. The SD-TRM and 9IM are equivalent in their representations of the topological relations between regions. Although the 9IM, which is an intersection model, and the SD-TRM, which is a distance-based model, are two completely different models, both models have the same level of detail. If all the nine values of the 9IM are computed, then the topological relation of spatial objects can be concluded. Likewise, if all the sign of the four values of the SD-TRM are computed, then the topological relation of spatial objects can be concluded. Table 2 lists all of the eight topological relations between spatial regions represented by the SD-TRM and 9IM. Each topological relation represented by the SD-TRM corresponds to a unique topological relation represented by the 9IM (Table 2).

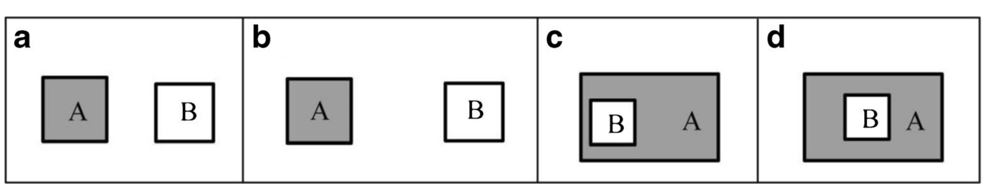


Fig. 5 Examples of the topological relations between two spatial regions. **a**, **b** Disjoint relations between spatial region A and B. **c**, **d** Contain relations between spatial region A and B

Table 1 Topological relation description by the 9IM, SP-9IM, SD-TRM and AD-TRM

	9IM	SP-9IM	SD-TRM	AD-TRM
Fig. 5a	$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0.9 & 0.9 & \infty \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 5 & 16 \\ 5 & 16 \end{bmatrix}$
Fig. 5b	$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0.9 & 0.9 & \infty \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 10 & 23 \\ 10 & 23 \end{bmatrix}$
Fig. 5c	$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0.4 & 0.8 \\ 0 & 0 & 1 \\ 0 & 0 & \infty \end{bmatrix}$	$\begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}$	$\begin{bmatrix} -14 & -1 \\ 1 & 14 \end{bmatrix}$
Fig. 5d	$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0.4 & 0.8 \\ 0 & 0 & 1 \\ 0 & 0 & \infty \end{bmatrix}$	$\begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}$	$\begin{bmatrix} -11 & -2 \\ 2 & 11 \end{bmatrix}$

From Table 2, it can be concluded that the SD-TRM and 9IM are equivalent in their representations of the eight topological relations. Due to this equivalence, the representations of the topological relations using the SD-TRM and 9IM can be directly converted to each other according to Table 2.

Third, the interoperability of the AD-TRM and 9IM is explored. As described above, the SD-TRM and 9IM are equivalent in their representations of the topological relations between spatial regions. The interoperability of the AD-TRM with the 9IM can be summarised in two steps: (1) The transformation from the AD-TRM to the SD-TRM can be expressed according to Formulas (11) and (2) the transformation from the SD-TRM to the 9IM is in accordance with Table 2. The relations in the AD-TRM can be converted into the 9IM after following these two steps. Because the SD-TRM and 9IM consider the sign of the distance and the content of the intersection, respectively, both models represent the topological relations with the same level of detail. Therefore, it is impossible to infer the AD-TRM from the 9IM.

Fourth, the interoperability of the SP-9IM with the AD-TRM, the SD-TRM and the 9IM is explored. Because the SP-9IM is a refinement of the 9IM, the value of the 9IM can be easily derived from the value of the SP-9IM. However, since the 9IM is the generalisation of the SP-9IM, and since the 9IM and the SD-TRM are equivalent, it is impossible to infer the SP-9IM from the 9IM or the SD-TRM. Due to the different emphases of the SP-9IM and AD-TRM on different aspects of metric properties, it is impossible to infer one model from the other.

The comparison of the topological relation representation between the AD-TRM and the comprehensive set of 11 metric refinements

A comprehensive set of 11 metric refinements were defined for the representation of the topological relations (Egenhofer and Dube 2009). In addition to the nine splitting measures, Egenhofer and Dube (2009) also defined two closeness measures. These two closeness measures are expansion closeness (EC) and contraction closeness (CC). For the EC, the buffer is normalised by the area after swelling (Formula (12)), while for the CC, the area of the reference region normalises this buffer zone to a value between 0 and 1 (Formula (13)). ΔA is the buffer from the boundary of A to the boundary of B in the Formulas (12–13). Both EC and CC convert an empty boundary-boundary intersection into a non-empty intersection.

$$\mathrm{EC} = \frac{\mathrm{area}\,(\Delta(A))}{\mathrm{area}\,(A) + \mathrm{area}\,(\Delta(A))} \tag{12}$$

$$\mathrm{CC} = \frac{\mathrm{area}\,(\Delta(A))}{\mathrm{area}\,(A)} \tag{13}$$

Although the distance can be expressed by EC and CC in some degree, there are some differences between EC/CC and AD-TRM in the expression of distance.

1. For meet relation, EC is zero, while the minimum and maximum distances from A to B and the minimum and maximum distances from B to A are not equal to zero in AD-TRM. For meet relation, the above mentioned distance can be expressed in AD-TRM.
2. For overlap relation, both EC and CC are zero, while all four values are not equal to zero in AD-TRM. For overlap relation, the above-mentioned distance can be expressed in AD-TRM.
3. For cover relation, both EC and CC are zero, while the minimum distance from A to B and the maximum distance from B to A are not equal to zero in AD-TRM. For cover relation, the above-mentioned distance can be expressed in AD-TRM.

Table 2 The equivalence of the topological relations represented by the SD-TRM and 9IM

	Disjoint	Meet	Overlap	Cover	Contain	Equal	CoveredBy	Inside
SD-TRM	$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$
9IM	$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

4. For coveredBy relation, both EC and CC are zero, while the maximum distance from A to B and the minimum distance from B to A are not equal to zero in AD-TRM. For coveredBy relation, the above-mentioned distance can be expressed in AD-TRM.
5. For inside relation, EC is greater than zero and less than 1, while all four values are not equal to zero in AD-TRM. The ratio and the distance are expressed in EC and AD-TRM, respectively. The ratio and the distance are different metric properties.
6. For contain relation, CC is greater than zero and less than 1, while all four values are not equal to zero in AD-TRM. The ratio and the distance are expressed in CC and AD-TRM, respectively. The ratio and the distance are different metric properties.

Even all the splitting measures and closeness measures are accounted for, there are many differences between the D-TRM and the comprehensive set of 11 metric refinements. The D-TRM can express more distance information than the comprehensive set of 11 metric refinements, whereas, the comprehensive set of 11 metric refinements can express more ratio information than the D-TRM.

Conclusions

The major novel contribution of this paper is its proposal of the D-TRM, which considers the topology and distance between two spatial regions within a unified framework. The AD-TRM, which focuses on the actual distance, and the SD-TRM, which concentrates on the sign of the distance, are two different subtypes of D-TRM. Eight topological relations, namely, disjoint, meet, overlap, cover, contain, equal, coveredBy, and inside, are effectively represented by the AD-TRM and the SD-TRM. A topological relation decision tree is introduced to prove the mutual exclusiveness among the eight topological relations represented by the SD-TRM. The results of a comparison between the 9IM, SP-9IM, AD-TRM and SD-TRM show the following: (1) the AD-TRM provides a greater level of detail than the 9IM; (2) the AD-TRM is the refinement of the SD-TRM, and the SD-TRM is the generalisation of the AD-TRM and (3) the SP-9IM and AD-TRM emphasise different aspects of metric properties. The result of a comparison of the topological relation representation between the AD-TRM and the comprehensive set of 11 metric refinements shows that the D-TRM can express more distance information than the comprehensive set of 11 metric refinements.

Although both the topology and distance are considered in the proposed model, this paper is primarily limited insomuch that only two-dimensional spatial regions are discussed and three-dimensional spatial objects are not included. When applied to the surface of a sphere, the equal and attach relations, the contain and embrace relations and the cover and entwined relations, cannot be distinguished by the D-TRM at present. Spatial objects embedded in a three-dimensional space and the surface of a sphere by the extended D-TRM will be discussed in a future investigation. In addition, this study addresses only spatial regions without holes, while other objects, such as point, line, spatial region with holes, multi-point, multi-line and multi-region objects, are not discussed. An extended D-TRM that can be used to describe the topological relations between these other objects will be the subject of future research. As the AD-TRM is used to describe the minimum and maximum distances between two regions, the composition of the topological relations based on the AD-TRM may be different from that based on the 9IM. Thus, the composition of the topological relations based on the AD-TRM will be studied in the future as well. The 9IM and other extended models based on these models have been widely used as bases for the standards of queries in spatial databases and for spatial reasoning, whereas the D-TRM requires extensive verification to prove its applicability in these areas.

Acknowledgments We appreciate the detailed suggestions and comments from the editor and the anonymous reviewers.

Funding The work described in this article was supported by the following research programs: The National Basic Research Program of China (973 Program) [grant number 2015CB954103], the National Natural Science Foundation of China [grant number 41622108, grant number 41301417, grant number 41701441], and the Priority Academic Program Development of Jiangsu Higher Education Institutions [grant number 164320H116].

References

Alboody A, Sedes F, Inglada J (2010) Modeling topological relations between uncertain spatial regions in geo-spatial databases: uncertain intersection and difference topological model. Second International Conference on Advances in Databases, Knowledge, and Data Applications, Menuires, France, pp 7–15

Allen J (1983) Maintaining knowledge about temporal intervals. Commun ACM 26(11):832–843

Bruns H, Egenhofer M (1998) Similarity of spatial scenes. In: Kraak J-M, Molenaar M (eds) Seventh international symposium on spatial data handling, Delft, the Netherlands. Taylor Francis, London, pp 173–184

Chen J, Li C, Li Z, Gold C (2001) A Voronoi-based 9-intersection model for spatial relations. Int J Geogr Inf Sci 15(3):201–220

Clementini E, Di Felice P, Oosterom PV (1993) A small set of formal topological relationships suitable for end-user interaction. International Symposium on Advances in Spatial Databases, Singapore, pp 277–295

Cohn A, Randell D, Cui Z (1995) Taxonomies of logically defined qualitative spatial relations. Int J Hum Comput Stud 43(5):831–846

Cohn A, Bennett B, Gooday J, Gotts N (1997) RCC: a calculus for region based qualitative spatial reasoning. Geoinformatica 1(3):275–316

Deng M, Cheng T, Chen X, Li Z (2007) Multi-level topological relations between spatial regions based upon topological invariants. Geoinformatica 11(2):239–267

Dong T (2005) Recognizing variable spatial environments—the theory of cognitive prism, PhD thesis, University of Bremen, Germany, pp 49–81

Dube M (2017) Topological augmentation: a step forward for qualitative partition reasoning. J Spat Inf Sci 14(1):1–29

Dube M, Barrett J, Egenhofer M (2015) From metric to topology: determining relations in discrete space. International Workshop on Spatial Information Theory Santa Fe, New Mexico, USA, pp 151–171

Duckham M, Worboys M (2001) Computational structure in three-valued nearness relations. In: Montello DR (ed) Spatial information theory. COSIT 2001. Lecture notes in computer science, vol 2205. Springer, Berlin, pp 76–91

Dutta S (1989) Qualitative spatial reasoning: a semi-quantitative approach using fuzzy logic. Design and implementation of large spatial databases, first symposium SSD'89, Santa Barbara. pp 345–364

Egenhofer M (1989) A formal definition of binary topological relationships. In: Litwin W, Schek H-J (eds) Third International Conference on Foundations of Data Organization and Algorithms (FODO) held in Paris, France. Springer-Verlag), Lecture Notes in Computer Science, New York, pp 457–472

Egenhofer M (2007) Temporal relations of intervals with a gap. International Symposium on Temporal Representation and Reasoning IEEE, pp 169–174

Egenhofer M, Dube M (2009) Topological relations from metric refinements. In ACM Sigspatial International Conference on Advances in Geographic Information Systems Seattle. pp 158–167

Egenhofer M, Franzosa R (1991) Point-set topological spatial relations. Int J Geogr Inf Sci 5(2):161–174

Egenhofer M, Franzosa R (1995) On the equivalence of topological relations. Int J Geogr Inf Sci 8(2):133–152

Egenhofer M, Herring J (1991) Categorizing binary topological relations between regions, lines, and points in geographic databases. http://www.spatial.cs.umn.edu/Courses/Spring10/8715/papers/MSD11_egenhofer_herring.pdf. Accessed 7 Dec 2018

Egenhofer M, Mark D (1995) Naive geography. In: Frank AU, Kuhn W (eds) Spatial information theory a theoretical basis for GIS. COSIT 1995. Lecture notes in computer science, vol 988. Springer, Berlin, p 13

Egenhofer M, Shariff A (1998) Metric details for natural-language spatial relations. ACM T Inform Syst 16(4):321–349

Egenhofer M, Sharma J (1993) Topological relations between regions in R^2 and Z^2. Advances in spatial databases–third international symposium on large spatial databases, SSD'93, Singapore. In: Abel D, Ooi BC (eds) Lecture notes in computer science. Springer, Berlin, pp 316–336

Egenhofer M, Sharma J, Mark D (1993) A critical comparison of the 4-intersection and 9-intersection models for spatial relations: formal analysis. In Proceedings of the AutoCarto Conference, Minneapolis, pp 1–22

Egenhofer M, Clementini P, Di Felice P (1994) Topological relations between regions with holes. Int J Geogr Inf Sci 8(2):129–144

Formica A, Mazzei M, Pourabbas E, Rafanelli M (2017) Enriching the semantics of the directed polyline-polygon topological relationships: the DLP-intersection matrix. J Geogr Syst 19(2):175–196

Frank A (1992) Qualitative spatial reasoning about distances and directions in geographic space. J Visual Lang Comput 3(4):343–371

Freemana J (1975) The modelling of spatial relations. Comput Graph Image Process 4(2):156–171

Gahegan M (1995) Proximity operators for qualitative spatial reasoning. In Proceedings of the international conference COSIT'95, Semmering, Austria, pp 31–44

Godoy F, Rodríguez A (2002) A quantitative description of spatial configurations. In: Advances in spatial data handling. Springer, Berlin, pp 299–311

Gotts N, Gooday J, Cohn A (1996) A connection based approach to common-sense topological description and reasoning. Monist 79(1):51–75

Guesgen H (2002) Reasoning about distance based on fuzzy sets. Appl Intell 17(3):265–270

Hernández D, Clementini E, Di Felice P (1995) Qualitative distances. Spatial information theory. A theoretical basis for GIS, 988, pp 45–57

Hornsby K, Egenhofer M, Hayes P (1999) Modeling cyclic change. Workshops on evolution and change in data management, reverse engineering in information systems, and the world wide web and conceptual modeling, vol 1727. Springer, Berlin, pp 98–109

Jonsson P, Drakengren T (1997) A complete classification of tractability in RCC-5. J Artif Intell Res 6(1):211–221

Kor A, Bennett B (2013) Syllogistic reasoning for cardinal direction relations. In: Proceedings of the 2013 international conference on artificial intelligence. Csrea press, pp 255–261

Kurata Y (2009) From three-dimensional topological relations to contact relations. In: Developments in 3D geo-information sciences. Springer, Berlin, pp 123–142

Leng L, Yang G, Chen S (2017) A combinatorial reasoning mechanism with topological and metric relations for change detection in river planforms: an application to globeland30's water bodies. ISPRS Int J Geo-Inf 6(1):1–13

Li S, Li Y (2006) On the complemented disk algebra. J Logic Algebraic Program 66(2):195–211

Liu K, Shi W (2007) Extended model of topological relations between spatial objects in geographic information systems. Int J Appl Earth Obs Geoinf 9(3):264–275

Nedas K, Egenhofer M (2008) Spatial—scene similarity queries. Trans GIS 12(6):661–681

Nedas K, Egenhofer M, Wilmsen D (2007) Metric details of topological line–line relations. Int J Geogr Inf Sci 21(1):21–48

Open GIS Consortium, Inc (1999) OpenGIS simple features specification for SQL (Revision 1.1). http://www.opengis.org/techno/specs/99-049.pdf. Accessed 7 Dec 2018

Penna GD, Magazzeni D, Orefice S (2017) A formal framework to represent spatial knowledge. Knowl Inf Syst 51:1–28

Randell D, Cohn A (1989) Modelling topological and metrical properties of physical processes. In Proceedings of the international conference on principles of knowledge representation and reasoning, Toronto, pp 357–368

Randell D, Cui Z, Cohn A (1992) A spatial logic based on regions and connection. In Proceedings of the 3rd international conference on principles of knowledge representation and reasoning, Morgan Kaufmann publishers Inc.: San Francisco, pp 165–176

Schneider M, Behr T (2006) Topological relationships between complex spatial objects. ACM T Database Syst 31(1):39–81

Schultz C, Guesgen H, Amor R (2007) A system for querying with qualitative distances in networks. IEEE International Fuzzy Systems Conference, London, UK, pp 1–6

Shariff A, Egenhofer M, Mark D (1998) Natural-language spatial relations between linear and areal objects: the topology and metric of English-language terms. Int J Geogr Inf Sci 12(3):215–246

Sharma J (1996) Integrated spatial reasoning in geographic information systems: combining topology and direction, PhD thesis. The University of Maine, Orono, pp 68–100

Shen J, Zhou T, Chen M (2017) A 27-intersection model for representing detailed topological relations between spatial objects in two-dimensional space. ISPRS Int J Geo-Inf 6(2):1–16

Sridhar M, Cohn A, Hogg D (2011) From video to RCC8: exploiting a distance based semantics to stabilise the interpretation of mereotopological relations. In: Egenhofer M, Giudice N, Moratz R, Worboys M (eds) Spatial information theory. COSIT 2011. Lecture Notes in Computer Science, vol 6899. Springer, Berlin, pp 110–125

Stell J, Worboys M (1997) The algebraic structure of sets of regions. In: SC Hirtle, AU Frank (eds) Spatial information theory a theoretical basis for GIS. COSIT 1997. Lecture notes in computer science, vol 1329. Springer, Berlin, Heidelberg, pp 163–174

Worboys M (1992) A generic model for planar geographical objects. Int J Geogr Inf Sci 6(5):353–372

Worboys M (2001) Nearness relations in environmental space. Int J Geogr Inf Sci 15(7):633–651